Oscillator on a train

(a) An accelerating frame of reference will have an extra force am opposing the direction of acceleration.

One can then work either with Force or potential:

<u>Force Method</u> $V(x) = kx^2/2$ (from question 1)

$$\Rightarrow F = -\frac{dV}{dx} = -kx$$

in accelerating frame:

$$F = -kx - am = -k(x - x_0)$$

with $x_0 = -\frac{am}{k}$

This is the same spring constant, just a different equilibrium point $\Rightarrow$ $\boxed{\text{Period is unchanged} = 1 \text{ second}}$

<u>Potential method</u>

- Force from acceleration can be written as potential amx

$\Rightarrow$ potential in accelerating frame

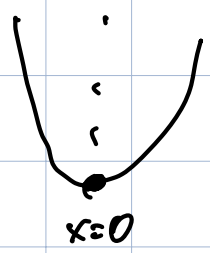
$$V(x) = \frac{kx^2}{2} + amx = \frac{k}{2}(x - x_0)^2 - \frac{kx_0^2}{2}$$

where $x_0 = -\frac{am}{k}$ (completing the square)

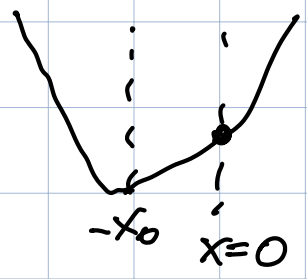
Same potential well, shifted equilibrium point $\Rightarrow$ $\boxed{\text{period is unchanged} = 1 \text{ second}}$

(b) Using potential method:

Before acceleration



With acceleration



$\therefore$ Amplitude of oscillations is from 0 to $2x_0$

Note: "Amplitude" is ambiguous and could mean x_0 or $2x_0$ – marks should be awarded for either answer so long as it is clearly written what they mean by amplitude

$$\boxed{x_0 = \frac{am}{k} = 1.3\text{cm}}$$

$a = 0.5\,\text{ms}^{-2}$

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m/k = \left(\frac{T}{2\pi}\right)^2 = 0.025$

Students can quote this formula, easy to derive if they need to.

(c) The acceleration will take 60s which is an integer multiple of the period.

$\Rightarrow$ at $t = 60\text{s}$, $x(t) = 0$

as it will be at its starting position after each full period.

The acceleration will now be zero, so it is once more at the bottom of the well $\Rightarrow$ $\boxed{\text{amplitude now} = 0}$

(d) • One might (initially) think the acceleration is pushing energy into the oscillating mass, so there will be some conservation law to give the final energy.

• From the result of part (c), this is clearly not true. The crucial point is that the mass is oscillating – so the acceleration could be giving it energy or taking it out depending on relative directions.

• While the acceleration does matter a bit, the more important quantity is <u>where</u> in the oscillation cycle the mass is when the acceleration stops.

  • Minimum amplitude if accelerating period is integer number of periods

  • Maximum amplitude if accelerating period is half-integer number of periods.

• Point for any sensible (and correct) discussion of some of the above.