You are a group of scientists on an space station called *Cheesecake* which orbits a star named *Blueberry*.

You have made a shocking discovery that in 100 million years a foreign star will collide with your own and destroy your civilisation. You know that there is a 'safe zone' five light years away and decide that the best course of action is to move the star rather than leaving the station. The government agrees to let you build a device which will move the star but they want the costs and the impact on the civilisation's daily lives as well to remain as low as possible.

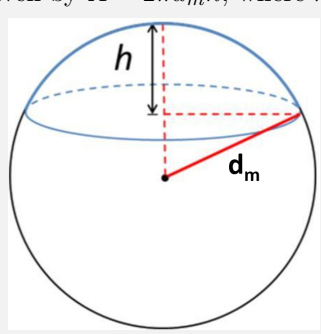
The device will use the momentum imparted by stellar photons to move the star and in turn your civilisation. The device will be made of *reflectonium*, an ultra-thin reflecting material, which your team will shape into a parabolic mirror and place a distance d_m from the star. When the photons hit the mirror they will be reflected having imparted momentum, hitting and leaving the surface.

Your team christens the device the *big-shiny-moving-star-inator* (or on Earth a *Shkadov thruster*), it is a statite meaning it will not orbit the star but remain at a fixed position (due to a balance of gravity and radiation pressure).

Useful data:

- Blueberry's peak spectrum wavelength, λ_{max}= 450 nm
- Blueberry's mass, $M = 4 \times 10^{31}$ kg
- Blueberry's radius, $R = 1.75 \times 10^{10}$ m
- Blueberry-to-inator distance, $d_m = 2 \times 10^{10}$ m
- Stefan-Boltzmann constant, $\sigma = 5.6704 \times 10^{-8}$ Wm^{-2}K^{-4}
- Wein's law, $\lambda_{max} T_{eff} = 2.898 \times 10^{-3}$ m K
- One light year = 9.46×10^{15} m
- One million years = 3.1536×10^{13} s

The surface area of a spherical cap is given by $A = 2\pi d_m h$, where h is the height (or depth) of the mirror.



(a) (8 marks) Workout the surface area of the *big-shiny-moving-star-inator* that:

(i) (7 marks) meets the government's criteria.

(ii) (1 mark) would take the shortest time to travel,

State any assumptions you have made to get to your answer.

The *big-shiny-moving-star-inator* is a statite, it will remain at a fixed position rather than orbiting around the star.
The radiation pressure is given below,

$$\langle P_{radiation} \rangle = \begin{cases} \frac{I}{c} & \text{Perfect absorber} \\ \frac{2I}{c} & \text{Perfect reflector,} \end{cases}$$

where I is the intensity at the mirror (flux) and c is the speed of light.5

(b) (2 marks) Show that the position of the *big-shiny-moving-star-inator* will not affect its ability to be a statite.

Solutions

(a) (i) (7 marks)

We know power = force × average velocity, when the photons from the star hit the mirror they impart momentum onto the mirror which contributes a force in the positive direction, as the mirror is a perfect reflector then the photons as they reflect off the mirror also push away from the mirror again giving momentum and contributing to the thrust in the positive direction. As photons travel at a constant speed, c in the vacuum of space we have an equation that relates the force moving the star due to the photon momentum and the power of the star that is hitting the mirror,

$$P_{mirror} = F \times c.$$

Using Newton's second law we can substitute for the star's mass and acceleration, then using basic linear motion equations write the power as,

$$P_{mirror} = \frac{2Mxc}{t^2},$$

where x is the distance to the safe zone, M is the star's mass and t is the time taken to get there.

[Award 1 mark for reaching this conclusion via kinematics and Newton's laws]

The star has a power (Luminosity), L in the ideal situation where the mirror was a hemisphere capturing half the total luminosity the power at the mirror would equal the luminosity of the star due to the photons giving two times the thrust when they reflect off the perfect mirror.

Now we must work out how much of the total luminosity will be hitting the mirror. The area of the mirror will be a fraction of the area of the flux sphere of radius d_m, $A_m = y \times A$. From this we can find a relationship between them,

$$y = \frac{A_m}{A} = \frac{2\pi d_m h}{4\pi {d_m}^2} = \frac{h}{2d_m}.$$

The luminosity of the star hitting the mirror can also be written as a ratio of the total luminosity $L_m = k \times L$, the value of k is given by,

$$k = \frac{L_m}{L} = \frac{FA_m}{FA} = y = \frac{h}{2d_m}.$$

[Award 1 mark for calculating ratios]

The power at the mirror will be double that of the luminosity that is hitting it,

$$P_{mirror} = 2 \times L_m = \frac{h}{d_m} L,$$

which we can now write in terms of the surface area of the mirror ($A_m = 2\pi d_m h$) yielding,

$$P_{mirror} = \frac{A_m L}{2\pi {d_m}^2}.$$

Now equating our two equations for the power at the mirror and making the area the subject, we get,

$$A_m = \frac{\pi {d_m}^2 Mxc}{t^2 L},$$

at this point we can do a quick sanity check and see that if the surface area of the mirror increased, the time taken would decease and the units all make sense.

[Award 2 marks for reaching this expression for the area of the mirror (1 mark for substitution and 1 mark for rearranging)]

Finally, we must work out the luminosity of the star, $L = F_{blackbody} \times A$ which can be found from the peak wavelength given in the question using Wein's law.

The flux of a blackbody is given by $F_{blackbodu} = \sigma {T_{effective}}^4$, where σ is the Stefan-Boltzmann constant, using Wein's law we arrive at an expression for the star's luminosity,

$$L = 4\pi R^2 \sigma \frac{{W_c}^4}{{\lambda_{max}}^4},$$

where we have written the proportionality constant of Wein's law as $W_c = 2.898 \times 10^{-3}$ m K.

Putting this expression into the the formula found for the surface area of the mirror in terms of the luminosity we get,

$$A_m = \frac{{d_m}^2 Mxc{\lambda_{max}}^4}{4t^2 R^2 \sigma {W_c}^4},$$

which we can substitute all the given values into and find the surface area of the mirror needed to move the star five light years in 100 million years is $A_m = 1.9107 \times 10^{17}$ m^2.

[Award final 3 marks for use of flux and luminosity expressions and then substituting to give final answer (2 for flux and luminosity and 1 for numerical answer)]

(ii) (1 mark)

Simply rearrange the formula found for the surface area to make time the subject,

$$t = \sqrt{\frac{{d_m}^2 Mxc{\lambda_{max}}^4}{4A_m R^2 \sigma {W_c}^4}},$$

we already know that time decreases with the area of the mirror, the maximum size the mirror could be to give the most thrust would be a hemisphere with $A_m = \frac{1}{2}A = \frac{1}{2}4\pi {d_m}^2$.

Putting in the appropriate values, if the mirror was a hemisphere around the star then the star would arrive at the safe zone in $t = 1.5198 \times 10^{13}$s which is approximately $482,000$ years.

[Award 1 mark for correct numerical answer (award half mark if correct rearranging to find t from A_m)]

(b) (2 marks)

The question says that a statite is a object that is static in space and not orbiting the star due to the balance of gravity and radiation pressure, very simply we equate the two forces.

First find out what the force of the radiation pressure is on the mirror,

$$\langle P_{radiation} \rangle = \frac{2I}{c} = \frac{2L}{4\pi c {d_m}^2},$$

using this we can write an expression for the force as $force = pressure \times cross - sectionalarea$.

Now, we equate the two forces $F_g = F_{rp}$,

$$\frac{GMm}{{d_m}^2} = \frac{L}{2\pi c{d_m}^2},$$

clearly it doesn't matter where the mirror is placed, it only depends on the mass of the star, mirror and the luminosity of the star.

[Award 1 mark for equating the two forces and writing an expression for them]

[Award 1 mark for explaining what this means, i.e. it clearly doesn't matter where it is placed as d_m cancels]