Solutions

Electron radius

The derivation is based on the relation between mass and its associate energy $W = mc^2$. For the electric field energy, it is sufficient to know the Coulomb's law expression. For any spherically symmetrical objects, the electric field is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{e}{r^2}.$$

We can use it to find the energy density of the electric field in the outer area

$$w = \frac{\varepsilon_0 E^2}{2}$$

If assuming electron as an evenly charged conductive sphere, the electric field is nonzero only outside the sphere. It allows finding the energy density distribution as

$$w(r) = \frac{\varepsilon_0 E^2}{2}$$

To get total energy, we integrate outside the electron "sphere"

$$W = \int_{\text{out}} w dV = 4\pi\frac{\varepsilon_0}{2}\int_{r_e}^{\infty}\left(\frac{1}{4\pi\varepsilon_0}\frac{e}{r^2}\right)^2 r^2 dr =$$

We can apply spherical coordinates and automatically integrate over angular coordinates getting the 4π factor. It brings us to

$$= \frac{e^2}{8\pi\varepsilon_0}\int_{r_e}^{\infty}\frac{dr}{r^2} = \frac{e^2}{8\pi\varepsilon_0 r_e}.$$

It brings us expression for the "electic" electron radius as (5 marks)

$$r_e = \frac{e^2}{8\pi\varepsilon_0 m_e c^2} = 1.4 \times 10^{-15} \text{ m}.$$

On the other hand, the equality between energies $m_e c^2 = \frac{e^2}{8\pi\varepsilon_0 r}$ allows calculating effective mass electric field outside of any sphere of interest with radius r. Therefore, for the second part, the effective electron "mass" can be found as (5 marks)

$$m_{eff} = \frac{e^2}{8\pi\varepsilon_0 r c^2} = 1.28 \times 10^{-29} \text{ kg}.$$