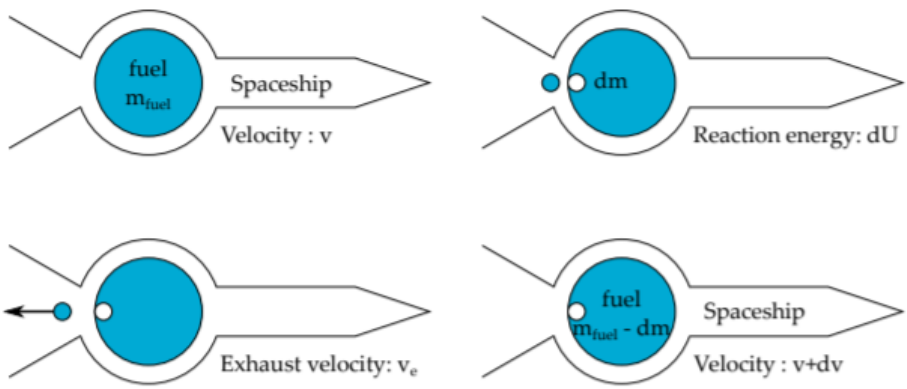
Solutions

Space craft

"it is impossible to perform interstellar travel with a standard combustion-based spaceship". To demonstrate this very broad statement, we will show that a combustion-based rocket can never reach velocities close to the speed of light. The main reason is that the embedded fuel contributes to the inertia of the system, making it very hard to propel, and that the mass-to-energy factor for chemical reactions is too weak. And since the closest star is already a few light-years away, a too low velocity forbids travelling between stars. We first derive the relation between the amount of fuel loaded in the ship, and the final velocity reached once all the fuel has been used.



We start the derivation of the relation of momentum conservation before and after a small portion of the fuel is exhausted (refer to the figure). We consider the Earth reference frame:

$$mv = (m - dm)(v + dv) + dm(v - v_e)$$

Opening brakes and neglecting the second-order term $dm \cdot dv$:

$$\cancel{mv} = \cancel{mv} - \cancel{dm\,v} + m\,dv - dm\,dv + \cancel{dm\,v} - dm\,v_e.$$

$$m\,dv = dm\,v_e \quad \rightarrow \quad dv = v_e \frac{dm}{m}$$

$$v_{final} = v_e \log \frac{m_0}{m} = v_e \log \frac{m_f + m_s}{m_s}.$$

It is an answer for part one (4 marks). This derivation can be refined by considering the portion-based exhaust. Assume that each take of the fuel is ϕs portion of the total amount: $\phi = m_{por}/m_f$

$$v_{final} = v_e \sum_{j=1}^{1/\phi} \frac{\phi}{1 - j\phi}.$$

We are making the estimate based on the number of atoms in the Universe. Let's assume that the spacecraft is as small as possible - a single atom, and the largest possible amount of the fuel - the entire content of the Universe. Substituting 10^{82} atoms, we get

$$v_{final} = v_e \log 10^{82} \approx 200 v_e.$$

Now we estimate the exhaust velocity. Having $q = 2.88 \times 10^5$ J/mol, we can relate the rest energy and the combustion-releasable one.

$$W_{comb} = \alpha\, mc^2 = q\nu = q\frac{m}{M} = \frac{mv_e^2}{2}.$$

$\alpha = \frac{q}{Mc^2}$ stands for ration between combustion and total energy. For hydrogen, it is equal to

$$\alpha = \frac{2.88 \times 10^5}{2 \times 10^{-3}} \approx 1.6 \times 10^{-9}.$$

In this way we derive exhaust velocity in terms of the speed of light: $v_e = c\sqrt{2\alpha}$. The final velocity is estimated as:

$$v_{final} = c\sqrt{2\alpha} \times \log 10^{82} \approx 0.01c.$$

Making the extremely upper bound estimate, we cannot reach relativistic velocities (even to one percent of the speed of light), and interstellar travels are not practical with combustion engines. This stands for part two (4 marks).

For part three (2 marks), we need to re-estimate α and q to get $v_f = 0.5c$. Having the expression above,

$$\alpha \to \frac{1}{2}\left(\frac{v_e/c}{\log 10^{82}}\right)^2 = \frac{1}{2}\left(\frac{0.5}{200}\right)^2 = 3.125 \times 10^{-6}.$$

Therefore, the required reaction energy is given as

$$q \to \alpha Mc^2 = 3.125 \times 10^{-6} \cdot 2 \times 10^{-3} \cdot 9 \times 10^{16} \approx 5.6 \times 10^8 \text{ J/mol}.$$