Solutions

Droplet physics

For part one (5 marks), the only force of consideration is surface tension, characterised by the surface tension parameter σ measured in [N/m]. Droplet inertia is characterised by liquid's density ρ [kg/m^3] and its radius [m] (larger droplet is more "resistant" to oscillations). Their combination with unknown powers should give a value of frequency ν, measured in [Hz=1/s]:

$$\nu = \sigma^A \cdot \rho^B \cdot r^C$$

$$\frac{1}{s} = \left(\frac{N}{m}\right)^A \cdot \left(\frac{kg}{m^3}\right)^B \cdot m^C = \left(\frac{kg}{s^2}\right)^A \cdot \left(\frac{kg}{m^3}\right)^B \cdot m^C$$

Let us collect powers of SI units from l.h.s. and r.h.s. If reducing the expression to basic SI units, it brings us three *independent equations*:

$$\text{kg:}\quad A + B = 0 \Rightarrow B = -\frac{1}{2}$$

$$\text{s:}\quad -1 = -2A \Rightarrow A = \frac{1}{2}$$

$$\text{m:}\quad -3B + C = 0 \Rightarrow C = 3B = -\frac{3}{2}$$

Combining these powers, one can get:

$$\nu \sim \sqrt{\frac{\sigma}{\rho r^3}}.$$

For part two (5 marks), the droplet radius r is measured in [m], and determined by a balance of surface tension which effectively "sticks" the droplet to the ceiling, and gravity, which pulls it down. The surface tension is characterised by σ in [N/m], and the gravity is characterised by the liquid's density ρ [kg/m^3] and acceleration of gravity g [m/s^2]. Similarly to the previous part, let us express radius as a product of all parameters in unknown powers:

$$r = \sigma^A \cdot \rho^B \cdot g^C$$

$$m^1 = kg^A \cdot s^{-2A} \cdot kg^B \cdot m^{-3B} \cdot m^C \cdot s^{-2C}.$$

In this way one can arrive for three equations for unknown A, B and C:

$$\text{kg:}\quad A + B = 0 \Rightarrow B = -A$$

$$\text{s:}\quad -2A - 2C = 0 \Rightarrow C = -A = B$$

$$\text{m:}\quad 1 = -3B + C = -2B \Rightarrow B = -\frac{1}{2} = C,\ A = \frac{1}{2}.$$

So, it brings us the final expression for radius:

$$r \sim \sigma^{1/2} \cdot \rho^{-1/2} \cdot g^{-1/2} \sim \sqrt{\frac{\sigma}{\rho g}}.$$